Entry Task: Differentiate

$$A)\ y = 5x^3 - \frac{x^{10}}{2}$$

$$B)\ y = x\sqrt{x} - 6 + \frac{3}{\sqrt{x}} \quad = x^1 x^{1/2} - 6 + 3x^{-1/2}$$

$$C)\ y = 5(2x)^3 \quad = 5 \cdot 8 \cdot x^3$$

$$D)\ y = 2\left(\frac{x^2}{3}\right)\left(\frac{9}{x^4}\right)x^3 \quad = 2 \cdot \frac{9}{3} \cdot \frac{x^2 x^3}{x^4} = 6x$$

$\boxed{A}$ $y = 5x^3 - \frac{1}{2}x^{10} \Rightarrow \boxed{\frac{dy}{dx} = 15x^2 - 5x^9}$

$\boxed{B}$ $y = x^{3/2} - 6 + 3x^{-1/2} \Rightarrow \boxed{\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 0 - \frac{3}{2}x^{-3/2}}$

$\boxed{C}$ $y = 40x^3 \Rightarrow \boxed{\frac{dy}{dx} = 120x^2}$

$\boxed{D}$ $y = 6x \Rightarrow \boxed{\frac{dy}{dx} = 6}$

ecall: *Power, Sum, Coeff. Rules*

$$\frac{d}{dx}(x^n) = n\,x^{n-1}.$$

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x).$$

$$\frac{d}{dx}\big(cf(x)\big) = cf'(x).$$

teps: 1. Expand.

2. Rewrite Powers.

3. Use power rule.

.5: Marginal Analysis and the roduct/Quotient Rules

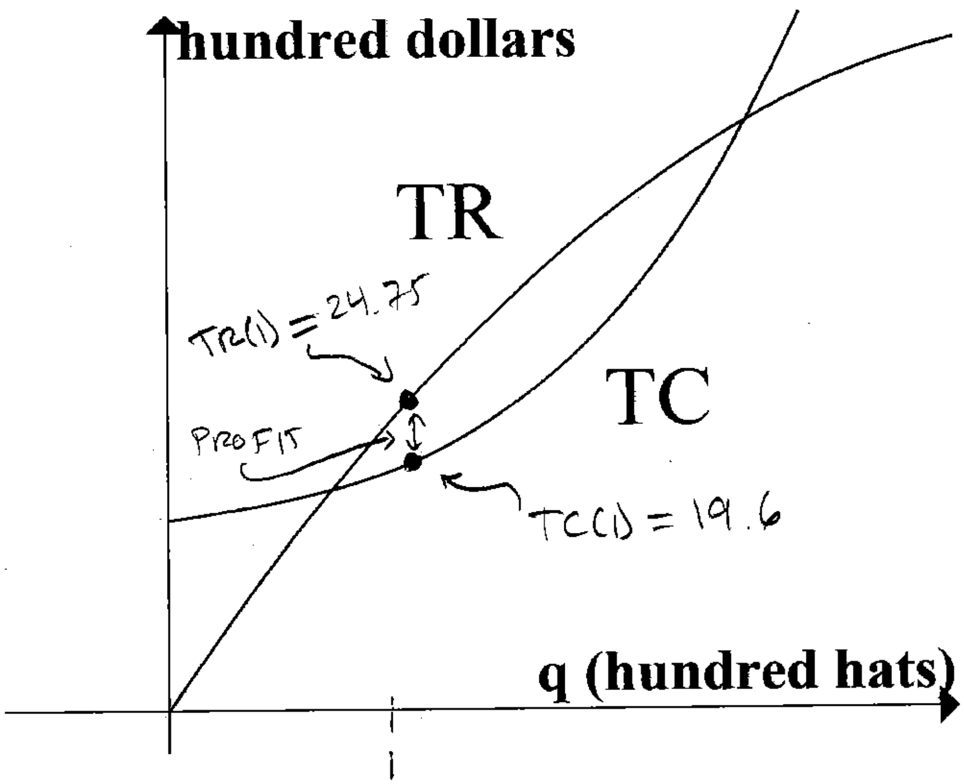
xample: Your company produces and ;ells hats. Based on data from recent nonths, you estimate:

'otal Revenue:

$$TR(q) = -3.75q^2 + 28.5q$$

'otal Cost:

$$TC(q) = 2q^3 - 0.4q^2 + 3q + 15$$



These functions give revenue and cost in a month where you produce

q hundred hats

$TR(q), TC(q)$ are in hundred dollars

Quick Review Question:
What is TR(0)? What is TC(0)?
What do these represent?

$TR(0) = -3.75(0)^2 + 28.5(0) = 0$

$TC(0) = 15$ hundred dollars

If you sell 0 hundred hats your revenue is 0 hundred dollars

If you produce 0 hundred hats your cost is 15 hundred dollars ($1500) *lumerical Review*:

Vhen q = 1 hundred hats (100 hats):

$$\text{ᒣ}R(1) = -3.75(1)^2 + 28.5(1)$$
$$= 24.75 \quad \text{hundred dollars}$$
$$\text{ᒣ}C(1) = 2(1)^3 - 0.4(1)^2 + 3(1) + 15$$
$$= 19.6 \quad \text{hundred dollars}$$

$$\text{'rofit} = P(1) = TR(1) - TC(1)$$
$$= 24.75 - 19.6$$
$$= 5.15 \text{ hundred dollars}$$

'hus, if you produce and sell
:xactly 100 hats this month,
hen your profit will be $515.

$$\underbrace{\begin{array}{l}\text{100 hats} \Leftrightarrow \text{TR of \$2475} \\ \text{100 hats} \Leftrightarrow \text{TC of \$1960}\end{array}}_{\text{PROFIT \$515}}$$

New Definition

In Math 112, we are going to redefine Marginal Revenue and Marginal Cost as follows:

$$MR(q) = TR'(q)$$
$$MC(q) = TC'(q)$$

Q: Using this new definition and our derivative rules, find MR(q) and MC(q).

$$TR(q) = -3.75q^2 + 28.5q$$

$$MR(q) = -3.75 \cdot 2q + 28.5$$

$$\boxed{MR(q) = -7.5q + 28.5}$$

$$TC(q) = 2q^3 - 0.4q^2 + 3q + 15$$

$$\boxed{MC(q) = 6q^2 - 1.2q + 3}$$

Q: What is MR(2)?
What is MC(2)?
What are the units?

$$MR(2) = -7.5(2) + 28.5 = 13.5$$

$$MC(2) = 6(2)^2 - 0.8(2) + 3 = 25.4$$

$$\text{slope units} = \frac{\cancel{\text{hundred}}\ \text{dollars}}{\cancel{\text{hundred}}\ \text{hats}} = \frac{\text{dollars}}{\text{hat}}$$

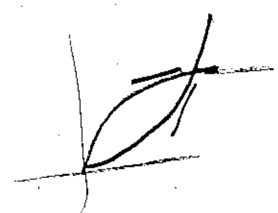
$$MR(2) = \$13.50\ \frac{\text{dollars}}{\text{hat}}$$

$$MC(2) = \$25.40\ \frac{\text{dollars}}{\text{hat}}$$

$\Rightarrow$ TC is "steeper" than TR at 2 hundred hats.

$\Rightarrow$ PROFIT DECREASING

WOULD YOU RATHER PRODUCE/SELL 200 OR 201?



omparing the Math 111 and 112 efinitions of MR and MC

ʌath 111:

MR(2) = change in revenue you go from selling 200 to 201 hats

= TR(2.01) – TR(2)

= 0.134625 hundred dollars (13.46 dollars)

ΛC(2) = change in cost you go from making 200 to 201 hats

= TC(2.01) – TC(2)

= 0.255162 hundred dollars (25.52 dollars)

ʌath 112:

MR(2) = TR'(2) = 13.50 dollars/hat

MC(2) = TC'(2) = 25.40 dollars/hat

NOTES:

1. The Math 111 and Math 112 definitions for MR and MC differ by 5 and 12 cents, respectively. (they are *close* to the same).
2. The derivative definition is much easier to compute and use.
3. We use the Math 111 definition to interpret and analyze our results.

NOTE IN GENERAL, THE DERIVATIVE IS APPROXIMATELY EQUAL TO THE CHANGE OVER THE "NEXT ITEM".

From HW: $q = \frac{108}{\sqrt{p}} - 1$

q = UNITS

p = PRICE

$q = 108p^{-1/2} - 1$

$q'(p) = -54p^{-3/2}$

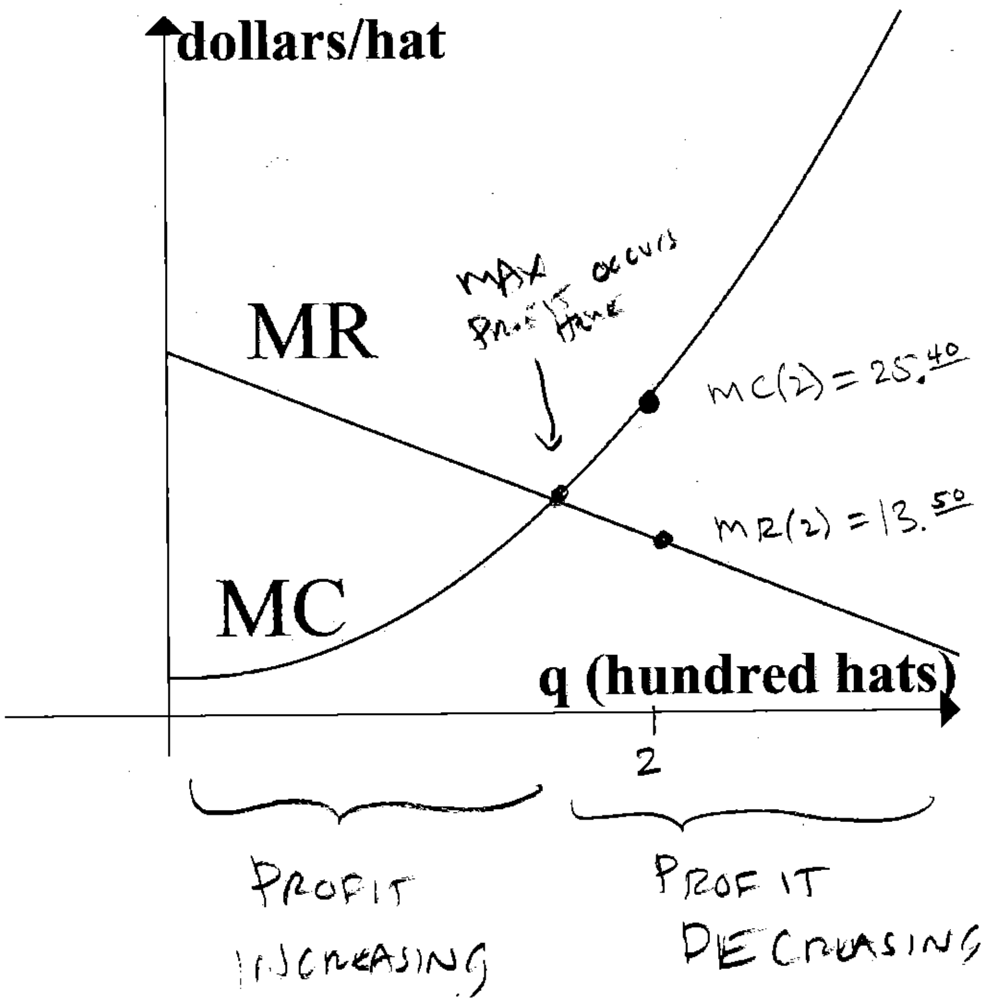
≈ CHANGE IN UNITS IF PRICE IS INCREASED BY ONE.

Marginal Revenue:

$$MR(q) = -7.5q + 28.5$$

Marginal Cost:

$$MC(q) = 6q^2 - 0.8q + 3$$



larginal Analysis

ecall a big observation
om Math 111:

$MR(q) > MC(q)$, then profit is
icreasing at q.
Revenue has a higher slope than cost)

$MR(q) < MC(q)$, then profit is
ecreasing at q.
Revenue has a lower slope than cost)

hus, profit is maximized at the
uantity when it switches from
$MR(q) > MC(q)$ to $MR(q) < MC(q)$

n other words, **profit is maximized**
vhere $MR(q) = MC(q)$.
match slopes)

n our example:

$$TR(q) = -3.75q^2 + 28.5q$$
$$TC(q) = 2q^3 - 0.4q^2 + 3q + 15$$

o **maximize profit**:

tep 1: Find MR(q) and MC(q):

$$MR(q) = TR'(q) = -7.5q + 28.5$$
$$MC(q) = TC'(q) = 6q^2 - 0.8q + 3$$

tep 2: Solve $MR(q) = MC(q)$

$$-7.5q + 28.5 \overset{?}{=} 6q^2 - 0.8q + 3$$
$$+7.5q \quad -28.5$$

$$\Rightarrow \quad 0 = \underset{\uparrow\, a}{6}q^2 + \underset{\uparrow\, b}{6.7}q \underset{\uparrow\, c}{-25.5}$$

$$q = \frac{-6.7 \pm \sqrt{6.7^2 - 4(6)(-25.5)}}{2(6)} \approx \begin{array}{c} -2.69 \\ \text{or} \\ 1.58 \end{array}$$

hundred hats

Product I sell 158 hats this month to maximize profit hree more derivative rules.

PRODUCT RULE: $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x).$

$$(F \cdot S)' = FS' + F'S$$

Ex) $y = \underbrace{(x^2+5x)}_{F}\underbrace{(x^4-x^3+1)}_{S}$

$y' = \underbrace{(x^2+5x)}_{F}\underbrace{(4x^3-3x^2)}_{S'} + \underbrace{(2x+5)}_{F'}\underbrace{(x^4-x^3+1)}_{S}$

Ex) $y = \underbrace{x^3}_{F}\underbrace{(x^2+x^{10})}_{S} \longrightarrow$ OR EXPAND FIRST, $y = x^5 + x^{13}$

$y' = 5x^4 + 13x^{12}$

$y' = \underbrace{x^3}_{F}\underbrace{(2x+10x^9)}_{S'} + \underbrace{3x^2}_{F'}\underbrace{(x^2+x^{10})}_{S}$

SAME

CHECK

YES!

$2x^4 + 10x^{12} + 3x^4 + 3x^{12} = 5x^4 + 13x^{12}$

QUOTIENT RULE:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$\left(\frac{N}{D}\right)' = \frac{DN' - ND'}{D^2}$$

Ex)

$$y = \frac{9x^2 - x}{14x + 7} \begin{matrix} \leftarrow N \\ \leftarrow D \end{matrix} \qquad \frac{dy}{dx} = \frac{(14x+7)(18x-1) - (19x^2-x)(14)}{(14x+7)^2}$$

Ex)

$$y = \frac{4x + 2}{x^3}$$

OR

$$y = \frac{4x}{x^3} + \frac{2}{x^3} = 4x^{-2} + 2x^{-3}$$

$$y' = -8x^{-3} - 6x^{-4}$$

$$y' = \frac{x^3(4) - (4x+2) \cdot 3x^2}{x^6}$$

SAME!

CHECK

$$\frac{4x^3 - 12x^3 - 6x^2}{x^6} = \frac{-8x^3 - 6x^2}{x^6} = -8x^{-3} - 6x^{-4}$$

✓

ou try: Differentiate

$. \; y = x^2(x^3+1)$

XPANDING $\Rightarrow \quad y = x^5 + x^2$

$$\boxed{\frac{dy}{dx} = 5x^4 + 2x}$$

--- OR ---

PRODUCT RULE $\Rightarrow \quad \frac{dy}{dx} = \underbrace{x^2}_{F} \cdot \underbrace{3x^2}_{S'} + \underbrace{2x}_{F'} \underbrace{(x^3+1)}_{S}$

$$= 3x^4 + 2x^4 + 2x$$

$$= \boxed{5x^4 + 2x}$$

$. \; y = \frac{5}{x^3}$

REWRITING $\Rightarrow \quad y = 5x^{-3}$

$$\boxed{\frac{dy}{dx} = -15x^{-4}}$$

--- OR ---

UOTIENT ULE $\quad \frac{dy}{dx} = \frac{x^3 \cdot (0) - 5 \cdot 3x^2}{x^6}$

$$= \frac{-15x^2}{x^6} = \boxed{-15x^{-4}}$$

$3. \; y = (x^2 + 3x)(\sqrt{x} - 5x^3)$

$x^{1/2}$

EXPANDING $\Rightarrow \quad y = x^{2.5} - 5x^5 + 3x^{1.5} - 15x^4$

$$\boxed{y' = 2.5x^{1.5} - 25x^4 + 4.5x^{0.5} - 60x^3}$$

--- OR ---

PRODUCT RULE $\Rightarrow \quad y' = (x^2+3x)\left(\tfrac{1}{2}x^{-1/2} - 15x^2\right) + (2x+3)(x^{1/2} - 5x^3)$

$$= \tfrac{1}{2}x^{1.5} - 15x^4 + \tfrac{3}{2}x^{1/2} - 45x^3 + 2x^{1.5} - 10x^4 + 3x^{0.5} - 15x^3$$

$$= \boxed{2.5x^{1.5} - 25x^4 + 4.5x^{0.5} - 60x^3}$$

$4. \; y = \frac{x^5}{3x^3 - x^5}$

QUOTIENT RULE $\Rightarrow$

$$\boxed{y' = \frac{(3x^3 - x^5)5x^4 - x^5(9x^2 - 5x^4)}{(3x^3 - x^5)^2}}$$

$$= \frac{3x^7 - 5x^9 - 9x^7 + 5x^9}{(3x^3 - x^5)^2}$$

$$= \frac{-6x^7}{(3x^3 - x^5)^2}$$

CHAIN

~~QUOTIENT~~ RULE:

$$\frac{d}{dx}(f(g(x))) = f'\big(g(x)\big)f'(x)$$

Ex) $y = (3x^2+1)^4$

OUTSIDE $= (\quad)^4 \Rightarrow f(u) = u^4$

INSIDE $= 3x^2+1 \Rightarrow g(x) = 3x^2+1$

$\Big\}\ f(g(x)) = (3x^2+1)^4$

$u^4 \to 4u^3 \qquad 3x^2+1 \to 6x$

LEAVE INSIDE HERE

$$y' = 4u^3 \cdot 6x = \underbrace{4(3x^2+1)^3}_{\text{DERIV. OF ``OUTSIDE''}}\ \underbrace{6x}_{\text{DERIV. OF ``INSIDE''}}$$

Ex) $y = (6x + x^{14})^{100}$

$$y' = 100(6x + x^{14})^{99} \cdot (6 + 14x^{13})$$